Reference Sheet

Fraction Arithmetic

- $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
- To add and subtract fractions, you need a common denominator.

  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

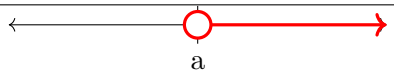
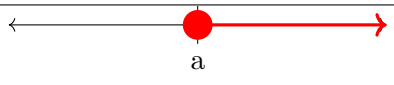
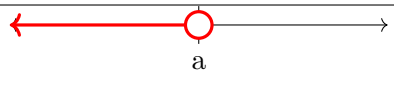
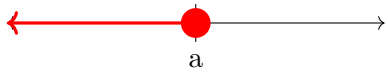
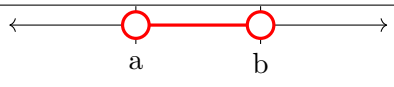
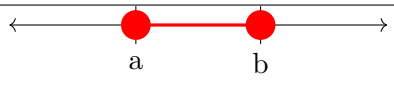
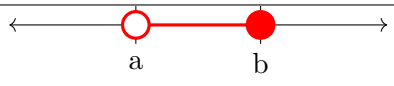
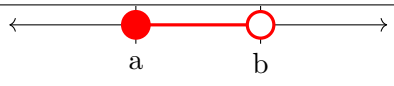
Miscellaneous

- To convert from a percentage to a decimal, divide the number by 100. This is equivalent to moving the decimal two places to the left. Do the opposite to convert from a decimal to a percentage.
- Distance between two points (x_1, y_1), (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint between two points (x_1, y_1), (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- $i = \sqrt{-1}$
- To divide complex numbers, write the division as a fraction and multiply the top and bottom by the conjugate of the denominator

Relations and Functions

- The domain of a relation/function is the set of all x-values
- The range of a relation/function is the set of all y-values
- To find x-intercepts, set $y = 0$ and solve. To find a y-intercept, set $x = 0$ and solve.
- Average rate of change of a function f from $x = a$ to $x = b$ is $\frac{f(b)-f(a)}{b-a}$
- Difference quotient of f is $\frac{f(x+h)-f(x)}{h}$
- Equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the center is (h, k) and the radius is r

Inequalities

Inequality	Number Line	Interval Notation
$x > a$		(a, ∞)
$x \geq a$		$[a, \infty)$
$x < a$		$(-\infty, a)$
$x \leq a$		$(-\infty, a]$
$a < x < b$		(a, b)
$a \leq x \leq b$		$[a, b]$
$a < x \leq b$		$(a, b]$
$a \leq x < b$		$[a, b)$

Reference Sheet Continued

Properties of Exponents

- $b^n b^m = b^{n+m}$
- $\dfrac{b^n}{b^m} = b^{n-m}$
- $b^0 = 1$
- $(b^n)^m = b^{nm}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $b^{-n} = \dfrac{1}{b^n}$
- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Lines/Linear Functions

- Standard/General Form: $Ax + By + C = 0$, where A and B aren't both 0
- Slope-Intercept Form: $y = mx + b$, where m is the slope and b is the y-intercept
- Point-Slope Form: $y - y_1 = m(x - x_1)$, where m is the slope and the point (x_1, y_1) is on the line
- $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Parallel lines have the same slope. Perpendicular lines have slopes that are opposite reciprocals of each other.

Quadratic Functions/Inequalities

- Equation of a parabola: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.
- The vertex of a parabola $f(x) = ax^2 + bx + c$ is located at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- The greatest common factor (GCF) is the product of the factors common to all terms in the polynomial.
- To complete the square for $x^2 + bx$: add $\left(\frac{b}{2}\right)^2$. If there is a number in front of your x^2, factor that out before completing the square.
- To factor $ax^2 + bx + c$ by grouping, find two numbers that multiply to $a \cdot c$ and add to b. Use these two numbers to split up the middle term bx.
- Factor by grouping: $a(b + c) + d(b + c) = (a + d)(b + c)$
- Difference of squares formula: $a^2 - b^2 = (a - b)(a + b)$
- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Transformations of graphs

- To graph a function by applying more than one transformation, (i.e. to graph $y = af(bx + c) + d$ using $y = f(x)$) use the following order:

  (1) Horizontal shifts using c (move left if $+c$ and right if $-c$)

  (2) Horizontal stretching/shrinking and/or reflecting across y-axis using b (divide all x-values by b)

  (3) Vertical stretching/shrinking and/or reflecting across x-axis using a (multiply all y-values by a)

  (4) Vertical shifts using d (move up if $+d$ and down if $-d$)

Reference Sheet Continued

Combining Functions

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$
- $(f \circ g)(x) = f(g(x))$
- $(g \circ f)(x) = g(f(x))$

General Functions:

- A function is odd if $f(-x) = -f(x)$
- A function is even if $f(-x) = f(x)$
- A turning point is a point where a graph switches from increasing to decreasing or vice versa.
- An inflection point is a point where a graph switches from concave up to concave down or vice versa.
- The leading term of a polynomial is the term of highest degree. The leading coefficient is the coefficient of the leading term.
- A polynomial of degree 0 or 1 is called linear. A polynomial with degree 2 is a quadratic, a polynomial with degree 3 is a cubic, and a polynomial with degree 4 is a quartic.
- The Rational Root Test: If x is a rational root/zero of a polynomial, then it can be written as $x = \frac{p}{q}$, where p is a factor of the constant term, and q is a factor of the leading coefficient.
- If $a + bi$ is a complex root/zero of a polynomial, then so is $a - bi$ and vice versa.
- The multiplicity of a root/zero is the degree of the factor associated with that root/zero
- A root/zero with an odd multiplicity will have a graph that goes through that number on the x-axis, a root/zero with even multiplicity will have a graph that bounces off of that number on the x-axis.
- To solve a polynomial equation:
  - Get everything to one side
  - Factor
  - Set each factor equal to zero and solve
- To solve a polynomial inequality:
  - Get everything to one side
  - Factor
  - Set each factor equal to zero and plot the resulting numbers on a number line
  - Test each number line segment and choose the pieces that satisfy the inequality

Reference Sheet Continued

Radicals & Rational Exponents

- $b^n b^m = b^{n+m}$
- $\dfrac{b^n}{b^m} = b^{n-m}$
- $b^0 = 1$
- $(b^n)^m = b^{nm}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $b^{-n} = \dfrac{1}{b^n}$
- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$
- $\sqrt[n]{x} = y \Rightarrow y^n = x$
- $\sqrt[n]{x} = x^{1/n}$
- $x^{m/n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$

- Solving a radical equation: Isolate the term with the radical (root sign), then raise both sides to the power that matches the radical. Combine like terms and solve.

Rational Functions

- A rational function is a function of the form $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomials
- Vertical asymptotes occur at the x values where the denominator equals 0
- For horizontal asymptotes:
  - If (degree of numerator)>(degree of denominator), then there is no horizontal asymptote
  - If (degree of numerator)=(degree of denominator), then the horizontal asymptote is $y = \frac{\text{lead coeff. of num.}}{\text{lead coeff. of denom.}}$
  - If (degree of numerator)<(degree of denominator), then the horizontal asymptote is $y = 0$
- If the degree of the numerator is exactly one more than the degree of the denominator, there is a slant asymptote. You find the equation of the slant asymptote by doing polynomial division
- Steps to solving a rational equation:
  (1) Either multiply all values by the least common denominator (LCD) **OR** get everything into one fraction on one side using the LCD
  (2) Factor if needed
  (3) Solve for x
- To solve a rational inequality:
  - Get everything to one side
  - Use a common denominator to get everything into one fraction (do NOT multiply by the LCD for inequalities)
  - Set each factor equal to zero and plot the resulting numbers on a number line
  - Test each number line segment and choose the pieces that satisfy the inequality