

Name: _____

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Express the intervals in set-builder notation and graph the intervals on a number lines.

(a) $[-1, 4]$

(b) $[5, 8]$

(c) $(6, \infty)$

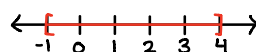
(d) $(-\infty, -2)$

(e) $(-\infty, 7)$

Solution

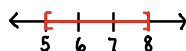
(a)

$$[-1, 4] = \{x : -1 \leq x \leq 4\}$$



(b)

$$[5, 8] = \{x : 5 \leq x \leq 8\}$$



(c)

$$(6, \infty) = \{x : x > 6\}$$



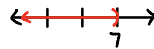
(d)

$$(-\infty, -2) = \{x : x < -2\}$$



(e)

$$(-\infty, 7) = \{x : x < 7\}$$



□

2. Find the following sets. Express them in interval notation.

(a) $(-5, 3) \cap [-4, 6]$

(b) $(-7, 6) \cap [0, 7]$

(c) $(-2, 3) \cup [0, 6]$

(d) $(-6, 5) \cup [-3, 6]$

(e) $[-1, \infty) \cap (3, \infty)$

Solution

(a) Thus the answer is $[-4, 3)$

(b) Thus the answer is $[0, 6)$

(c) Thus the answer is $(-2, 6]$

(d) Thus the answer is $(-6, 6]$

(e) Thus the answer is $(3, \infty)$

□

3. Use interval notation to express the solution set and graph the set on a number line if the solution set is not empty.

- (a) $5x + 3 > 23$
- (b) $2x + 4 > 10$
- (c) $-3x \geq 9$
- (d) $6(x + 1) + 4 \geq 5x + 17$
- (e) $7(x + 1) + 1 \geq 6x + 9$

Solution

(a)

$$\begin{aligned} 5x + 3 > 23 &\Leftrightarrow 5x > 20 \\ &\Leftrightarrow x > 4 \end{aligned}$$

Interval Notation: $(4, \infty)$



(b)

$$\begin{aligned} 2x + 4 > 10 &\Leftrightarrow 2x > 6 \\ &\Leftrightarrow x > 3 \end{aligned}$$

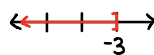
Interval Notation: $(3, \infty)$



(c)

$$-3x \geq 9 \rightarrow x \leq -3$$

Interval Notation: $(-\infty, -3]$



(d)

$$\begin{aligned} 6(x + 1) + 4 &\geq 5x + 17 \Leftrightarrow 6x + 6 + 4 \geq 5x + 17 \\ &\Leftrightarrow 6x + 10 \geq 5x + 17 \\ &\Leftrightarrow x \geq 7 \end{aligned}$$

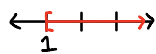
Interval Notation: $[7, \infty)$



(e)

$$\begin{aligned}7(x+1)+1 \geq 6x+9 &\Leftrightarrow 7x+7+1 \geq 6x+9 \\&\Leftrightarrow 7x+8 \geq 6x+9 \\&\Leftrightarrow x \geq 1\end{aligned}$$

Interval Notation: $\boxed{[1, \infty)}$



□

4. Solve the compound inequalities

(a) $-7 \leq x - 4 < -2$

(b) $-3 \leq x - 2 \leq 5$

(c) $5 < x + 5 < 8$

(d) $-3 < 2x - 1 < 5$

(e) $-2 \leq 3x + 7 \leq 13$

Solution

(a)

$$-7 \leq x - 4 < -2 \Leftrightarrow \boxed{-3 \leq x \leq 1}$$

(b)

$$-3 \leq x - 2 \leq 5 \Leftrightarrow \boxed{-5 \leq x \leq 7}$$

(c)

$$5 < x + 5 < 8 \Leftrightarrow \boxed{0 < x < 3}$$

(d)

$$\begin{aligned} -3 < 2x - 1 < 5 &\Leftrightarrow -2 < 2x < 6 \\ &\Leftrightarrow \boxed{-1 < x < 3} \end{aligned}$$

(e)

$$\begin{aligned} -2 \leq 3x + 7 \leq 13 &\Leftrightarrow -9 \leq 3x \leq 6 \\ &\Leftrightarrow \boxed{-3 \leq x \leq 2} \end{aligned}$$

□

5. Solve the following absolute inequalities. Graph the solution sets.

(a) $|x + 4| \leq 7$

(b) $|x - 6| \geq 2$

(c) $|2x - 2| > 6$

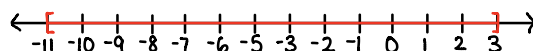
(d) $-3|x - 4| \geq -15$

(e) $6 > |5 - x|$

Solution

(a)

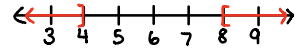
$$\begin{aligned} |x + 4| \leq 7 &\Leftrightarrow -7 \leq x + 4 \leq 7 \\ &\Leftrightarrow \boxed{-11 \leq x \leq 3} \end{aligned}$$



(b)

$$|x - 6| \geq 2 \Leftrightarrow x - 6 \geq 2 \text{ or } x - 6 \leq -2$$

$$\Leftrightarrow \boxed{x \geq 8 \text{ or } x \leq 4}$$

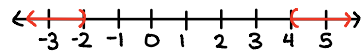


(c)

$$|2x - 2| > 6 \Leftrightarrow 2x - 2 > 6 \text{ or } 2x - 2 < -6$$

$$\Leftrightarrow 2x > 8 \text{ or } 2x < -4$$

$$\Leftrightarrow \boxed{x > 4 \text{ or } x < -2}$$

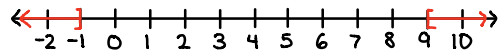


(d)

$$-3|x - 4| \geq -15 \Leftrightarrow |x - 4| \geq 5$$

$$\Leftrightarrow x - 4 \geq 5 \text{ or } x - 4 \leq -5$$

$$\Leftrightarrow \boxed{x \geq 9 \text{ or } x \leq -1}$$



(e)

$$6 > |5 - x| \Leftrightarrow |5 - x| < 6$$

$$\Leftrightarrow -6 < 5 - x < 6$$

$$\Leftrightarrow -11 < -x < 1$$

$$\Leftrightarrow \boxed{-1 < x < 11}$$

