Math 134 Spring 2018
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Classwork 19 Name: ______________________________

Work on as many problems as you can together with your group members. Towards the end of lecture your group will be asked to present problems correctly to receive classwork points.

1. Find $f(g(x))$ and $g(f(x))$ and determine whether f and g are inverses.

   (a) $f(x) = 8x - 9, g(x) = \dfrac{x+9}{8}$

   (b) $f(x) = 9x - 5, g(x) = \dfrac{x+9}{5}$

   (c) $f(x) = 7x - 6, g(x) = \dfrac{x+6}{7}$

   (d) $f(x) = 6x - 7, g(x) = \dfrac{x+7}{6}$

   (e) $f(x) = 9x + 5, g(x) = \dfrac{x+9}{5}$

   Solution

   (a)

$$\begin{aligned} f(g(x)) &= f\left(\frac{x+9}{8}\right) \\ &= 8\left(\frac{x+9}{8}\right) - 9 \\ &= x + 9 - 9 \\ &= x\checkmark \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(8x-9) \\ &= \frac{8x-9+9}{8} \\ &= \frac{8x}{8} \\ &= x\checkmark \end{aligned}$$

Thus f and g $\boxed{\text{are inverses.}}$

(b)

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{x+9}{5}\right) \\
&= 9\left(\frac{x+9}{5}\right) - 5 \\
&= \frac{9x+81}{5} - 5 \\
&= \frac{9x+106}{5} \neq x
\end{aligned}
$$

$$
\begin{aligned}
g(f(x)) &= g(9x-5) \\
&= \frac{9x-5+9}{5} \\
&= \frac{9x+4}{5} \neq x
\end{aligned}
$$

Thus f and g $\boxed{\text{are not inverses}}$

(c)

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{x+6}{7}\right) \\
&= 7\left(\frac{x+6}{7}\right) - 6 \\
&= x + 6 - 6 \\
&= x\checkmark
\end{aligned}
$$

$$
\begin{aligned}
g(f(x)) &= g(7x-6) \\
&= \frac{7x-6+6}{7} \\
&= \frac{7x}{7} \\
&= x\checkmark
\end{aligned}
$$

Thus f and g $\boxed{\text{are inverses.}}$

(d)

$$\begin{aligned}
f(g(x)) &= f\left(\frac{x+7}{6}\right) \\
&= 6\left(\frac{x+7}{6}\right) - 7 \\
&= x + 7 - 7 \\
&= x\checkmark
\end{aligned}$$

$$\begin{aligned}
g(f(x)) &= g(6x-7) \\
&= \frac{6x-7+7}{6} \\
&= \frac{6x}{6} \\
&= x\checkmark
\end{aligned}$$

Thus f and g $\boxed{\text{are inverses.}}$

(e)

$$\begin{aligned}
f(g(x)) &= f\left(\frac{x+9}{5}\right) \\
&= 9\left(\frac{x+9}{5}\right) + 5 \\
&= \frac{9x+81}{5} + 5 \\
&= \frac{9x+106}{5} \neq x
\end{aligned}$$

$$\begin{aligned}
g(f(x)) &= g(9x+5) \\
&= \frac{9x+5+9}{5} \\
&= \frac{9x+14}{5} \neq x
\end{aligned}$$

Thus f and g $\boxed{\text{are not inverses}}$

$\square$

2. Find an equation for the inverse function.

(a) $f(x) = x^3 + 4$

(b) $h(x) = (x+3)^5$

(c) $f(x) = \dfrac{13}{x}$

(d) $f(x) = \dfrac{x^3 - 1}{4}$

(e) $f(x) = x^{1/4}$

Solution

(a)

$$\begin{aligned} f(x) = x^3 + 4 \to y = x^3 + 4 \\ &\to x = y^3 + 4 \Leftrightarrow y = \sqrt[3]{x-4} \\ &\to \boxed{f^{-1}(x) = \sqrt[3]{x-4}} \end{aligned}$$

(b)

$$\begin{aligned} h(x) = (x+3)^5 \to y = (x+3)^5 \\ &\to x = (y+3)^5 \Leftrightarrow \sqrt[5]{x} = y + 3 \Leftrightarrow y = \sqrt[5]{x} - 3 \\ &\to \boxed{h^{-1}(x) = \sqrt[5]{x} - 3} \end{aligned}$$

(c)

$$\begin{aligned} f(x) = \frac{13}{x} \to y = \frac{13}{x} \\ &\to x = \frac{13}{y} \Leftrightarrow y = \frac{13}{x} \\ &\to \boxed{f^{-1}(x) = \frac{13}{x}} \end{aligned}$$

(d)

$$\begin{aligned} f(x) = \frac{x^3 - 1}{4} \to y = \frac{x^3 - 1}{4} \\ &\to x = \frac{y^3 - 1}{4} \\ &\Leftrightarrow 4x = y^3 - 1 \Leftrightarrow y = \sqrt[3]{4x+1} \\ &\to \boxed{f^{-1}(x) = \sqrt[3]{4x+1}} \end{aligned}$$

(e)

$$\begin{aligned} f(x) = x^{1/4} \to y = x^{1/4} \\ &\to x = y^{1/4} \Leftrightarrow y = x^4 \\ &\to \boxed{f^{-1}(x) = x^4} \end{aligned}$$

$\square$

3. Find $(f \circ g)^{-1}(x)$ and $g^{-1}(f^{-1}(x))$.

(a) If $f(x) = 9x$, $g(x) = x + 6$

(b) If $f(x) = 4x$, $g(x) = x - 5$

(c) If $f(x) = 3x$, $g(x) = 2x + 1$

(d) If $f(x) = 5x$, $g(x) = 3x + 4$

(e) If $f(x) = 8x$, $g(x) = x - 6$

Solution

(a)

$$(f \circ g)(x) = f(x + 6) = 9x + 54$$

$$\begin{aligned}(f \circ g)(x) = 9x + 54 \to y = 9x + 54 \\ &\to x = 9y + 54 \Leftrightarrow y = \frac{x - 54}{9} \\ &\to \boxed{(f \circ g)^{-1}(x) = \frac{x - 54}{9}}\end{aligned}$$

$$\begin{aligned}f(x) = 9x \to y = 9x \\ &\to x = 9y \Leftrightarrow y = \frac{x}{9} \\ &\to f^{-1}(x) = \frac{x}{9}\end{aligned}$$

$$\begin{aligned}g(x) = x + 6 \to y = x + 6 \\ &\to x = y + 6 \Leftrightarrow y = x - 6 \\ &\to g^{-1}(x) = x - 6\end{aligned}$$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x}{9}\right) = \frac{x}{9} - 6 = \boxed{\frac{x - 54}{9}}$$

(b)

$$(f \circ g)(x) = f(x-5) = 4x - 20$$

$$\begin{aligned}(f \circ g)(x) = 4x - 20 &\to y = 4x - 20\\ &\to x = 4y - 20 \Leftrightarrow y = \frac{x+20}{4}\\ &\to \boxed{(f \circ g)^{-1}(x) = \frac{x+20}{4}}\end{aligned}$$

$$\begin{aligned}f(x) = 4x &\to y = 4x\\ &\to x = 4y \Leftrightarrow y = \frac{x}{4}\\ &\to f^{-1}(x) = \frac{x}{4}\end{aligned}$$

$$\begin{aligned}g(x) = x - 5 &\to y = x - 5\\ &\to x = y - 5 \Leftrightarrow y = x + 5\\ &\to g^{-1}(x) = x + 5\end{aligned}$$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x}{4}\right) = \frac{x}{4} + 5 = \boxed{\frac{x+20}{4}}$$

(c)

$$(f \circ g)(x) = f(2x+1) = 6x+3$$

$$\begin{aligned}
(f \circ g)(x) = 6x+3 &\rightarrow y = 6x+3 \\
&\rightarrow x = 6y+3 \\
&\Leftrightarrow y = \frac{x-3}{6} \\
&\rightarrow \boxed{(f \circ g)^{-1}(x) = \frac{x-3}{6}}
\end{aligned}$$

$$\begin{aligned}
f(x) = 3x &\rightarrow y = 3x \\
&\rightarrow x = 3y \Leftrightarrow y = \frac{x}{3} \\
&\rightarrow f^{-1}(x) = \frac{x}{3}
\end{aligned}$$

$$\begin{aligned}
g(x) = 2x+1 &\rightarrow y = 2x+1 \\
&\rightarrow x = 2y+1 \Leftrightarrow y = \frac{x-1}{2} \\
&\rightarrow g^{-1}(x) = \frac{x-1}{2}
\end{aligned}$$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x}{3}\right) = \frac{\frac{x}{3}-1}{2} = \boxed{\frac{x-3}{6}}$$

(d)

$$(f \circ g)(x) = f(3x+4) = 15x+20$$

$$\begin{aligned}(f \circ g)(x) = 15x+20 &\to y = 15x+20 \\ &\to x = 15y+20 \Leftrightarrow y = \frac{x-20}{15} \\ &\to \boxed{(f \circ g)^{-1}(x) = \frac{x-20}{15}}\end{aligned}$$

$$\begin{aligned}f(x) = 5x &\to y = 5x \\ &\to x = 5y \Leftrightarrow y = \frac{x}{5} \\ &\to f^{-1}(x) = \frac{x}{5}\end{aligned}$$

$$\begin{aligned}g(x) = 3x+4 &\to y = 3x+4 \\ &\to x = 3y+4 \Leftrightarrow y = \frac{x-4}{3} \\ &\to g^{-1}(x) = \frac{x-4}{3}\end{aligned}$$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x}{5}\right) = \frac{\frac{x}{5}-4}{3} = \boxed{\frac{x-20}{15}}$$

(e)

$$(f \circ g)(x) = f(x-6) = 8x - 48$$

$$\begin{aligned}
(f \circ g)(x) = 8x - 48 &\to y = 8x - 48 \\
&\to x = 8y - 48 \Leftrightarrow y = \frac{x+48}{8} \\
&\to \boxed{(f \circ g)^{-1}(x) = \frac{x+48}{8}}
\end{aligned}$$

$$\begin{aligned}
f(x) = 8x &\to y = 8x \\
&\to x = 8y \\
&\Leftrightarrow y = \frac{x}{8} \\
&\to f^{-1}(x) = \frac{x}{8}
\end{aligned}$$

$$\begin{aligned}
g(x) = x - 6 &\to y = x - 6 \\
&\to x = y - 6 \Leftrightarrow y = x + 6 \\
&\to g^{-1}(x) = x + 6
\end{aligned}$$

$$g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{x}{8}\right) = \frac{x}{8} + 6 = \boxed{\frac{x+48}{8}}$$

$\square$

4. Given the function $f(x)$, find its inverse $f^{-1}(x)$, then graph f and f^{-1} on the same rectangular coordinate system. Lastly, Use interval notation to give the domain and the range of f and f^{-1}.

(a) $f(x) = (x-2)^2 - 1, x \geq 2$

(b) $f(x) = x^2 - 3, x \geq 0$

(c) $f(x) = (x+2)^2 + 1, x \geq -2$

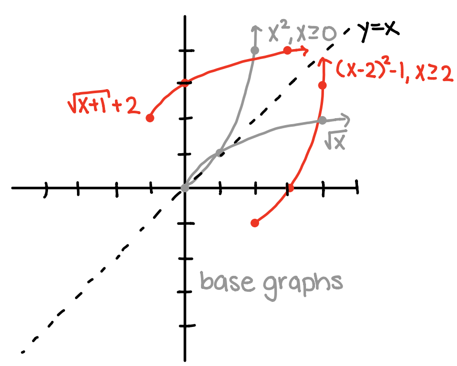
(d) $f(x) = (x-3)^2 + 1, x \geq 3$

(e) $f(x) = (x-3)^2, x \leq 3$

Solution

(a)

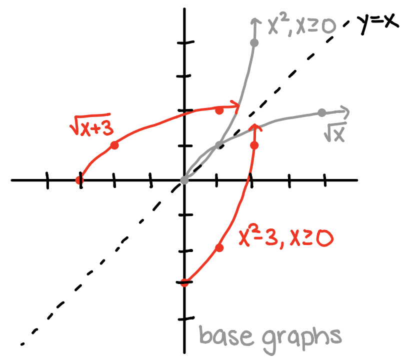
$$
\begin{aligned}
f(x) = (x-2)^2 - 1, x \geq 2 \rightarrow y &= (x-2)^2 - 1, x \geq 2 \\
&\rightarrow x = (y-2)^2 - 1, y \geq 2 \\
&\Leftrightarrow (y-2)^2 = x + 2, y \geq 2 \\
&\Rightarrow y - 2 = \pm\sqrt{x+1}, y \geq 2 \\
&\Rightarrow y = 2 \pm \sqrt{x+1}, y \geq 2 \\
&\Rightarrow y = 2 + \sqrt{x+1} \\
&\rightarrow \boxed{f^{-1}(x) = 2 + \sqrt{x+1}}
\end{aligned}
$$



$$\boxed{\text{Dom}(f) = \text{Ran}(f^{-1}) = [2, \infty) \text{ and } \text{Dom}(f^{-1}) = \text{Ran}(f) = [-1, \infty)}$$

(b)

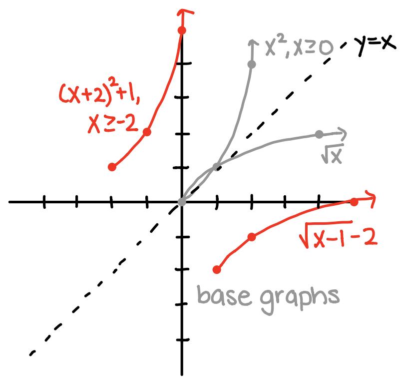
$$
\begin{aligned}
f(x) = x^2 - 3, x \geq 0 \rightarrow y = x^2 - 3, x \geq 0 \\
&\rightarrow x = y^2 - 3, y \geq 0 \\
&\Rightarrow y = \pm\sqrt{x+3}, y \geq 0 \\
&\Rightarrow y = \sqrt{x+3} \\
&\rightarrow \boxed{f^{-1}(x) = \sqrt{x+3}}
\end{aligned}
$$



$$\boxed{\text{Dom}(f) = \text{Ran}(f^{-1}) = [0, \infty) \text{ and } \text{Dom}(f^{-1}) = \text{Ran}(f) = [-3, \infty)}$$

(c)

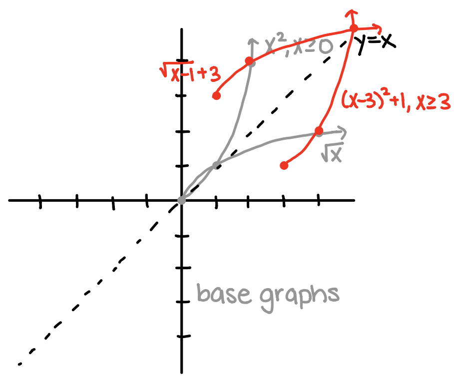
$$\begin{aligned} f(x) = (x+2)^2+1, x \geq -2 \rightarrow y &= (x+2)^2+1, x \geq -2 \\ \rightarrow x &= (y+2)^2+1, y \geq -2 \\ \Rightarrow (y+2)^2 &= x-1, y \geq -2 \\ \Rightarrow y+2 &= \pm\sqrt{x-1}, y \geq -2 \\ \Rightarrow y &= -2 \pm \sqrt{x-1}, y \geq -2 \\ \Rightarrow y &= -2+\sqrt{x-1} \\ \rightarrow & \boxed{f^{-1}(x) = -2+\sqrt{x-1}} \end{aligned}$$



$$\boxed{\text{Dom}(f) = \text{Ran}(f^{-1}) = [-2, \infty) \text{ and } \text{Dom}(f^{-1}) = \text{Ran}(f) = [1, \infty)}$$

(d)

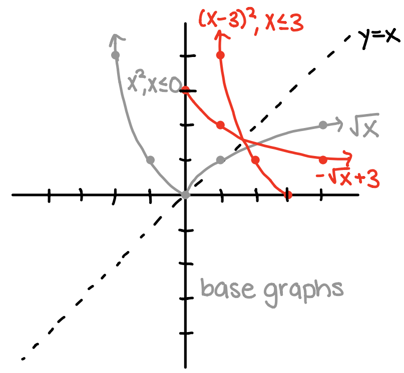
$$
\begin{aligned}
f(x) = (x-3)^2 + 1, x \geq 3 \rightarrow y &= (x-3)^2 + 1, x \geq 3 \\
\rightarrow x &= (y-3)^2 + 1, y \geq 3 \\
\Rightarrow (y-3)^2 &= x - 1, y \geq 3 \\
\Rightarrow y - 3 &= \pm\sqrt{x-1}, y \geq 3 \\
\Rightarrow y &= 3 \pm \sqrt{x-1}, y \geq 3 \\
\Rightarrow y &= 3 + \sqrt{x-1} \\
\rightarrow & \boxed{f^{-1}(x) = 3 + \sqrt{x-1}}
\end{aligned}
$$



$$\boxed{\text{Dom}(f) = \text{Ran}(f^{-1}) = [3, \infty) \text{ and } \text{Dom}(f^{-1}) = \text{Ran}(f) = [1, \infty)}$$

(e)

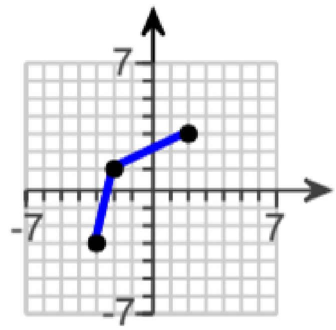
$$\begin{aligned} f(x) = (x-3)^2, x \le 3 \rightarrow y &= (x-3)^2, x \le 3 \\ \rightarrow x &= (y-3)^2, y \le 3 \\ \Rightarrow y - 3 &= \pm\sqrt{x}, y \le 3 \\ \Rightarrow y &= 3 \pm \sqrt{x}, y \le 3 \\ \Rightarrow y &= 3 - \sqrt{x} \\ \rightarrow & \boxed{f^{-1}(x) = 3 - \sqrt{x}} \end{aligned}$$



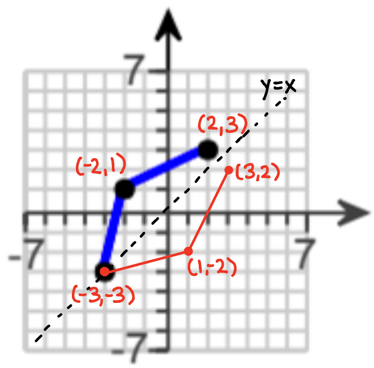
$$\boxed{\text{Dom}(f) = \text{Ran}(f^{-1}) = (-\infty, 3] \text{ and } \text{Dom}(f^{-1}) = \text{Ran}(f) = [0, \infty)}$$

$\square$

5. Use the graph below to draw a graph of its inverse function.



Solution



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